



Universität Hamburg  
DER FORSCHUNG | DER LEHRE | DER BILDUNG



MARTIN-LUTHER-UNIVERSITÄT  
HALLE-WITTENBERG



TECHNISCHE  
UNIVERSITÄT  
DRESDEN

Unterstützt von / Supported by



Alexander von Humboldt  
Stiftung/Foundation

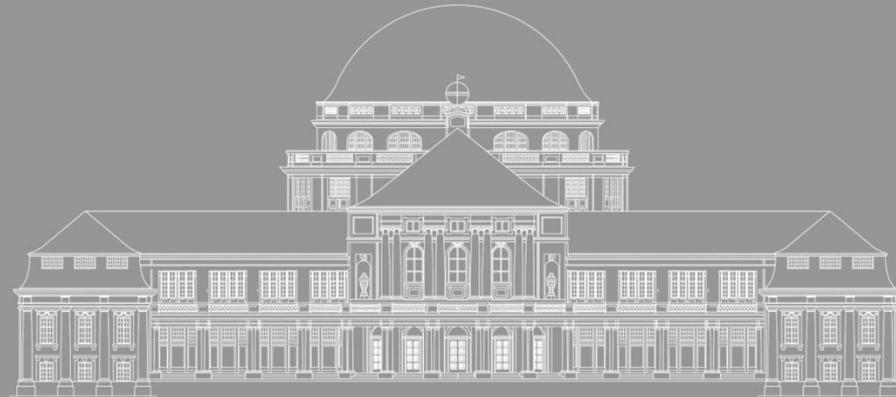
# *Quod erat demonstrandum:*

## Understanding and explaining equations in physics teacher education

Ricardo Karam<sup>1</sup> & Olaf Krey<sup>2</sup>

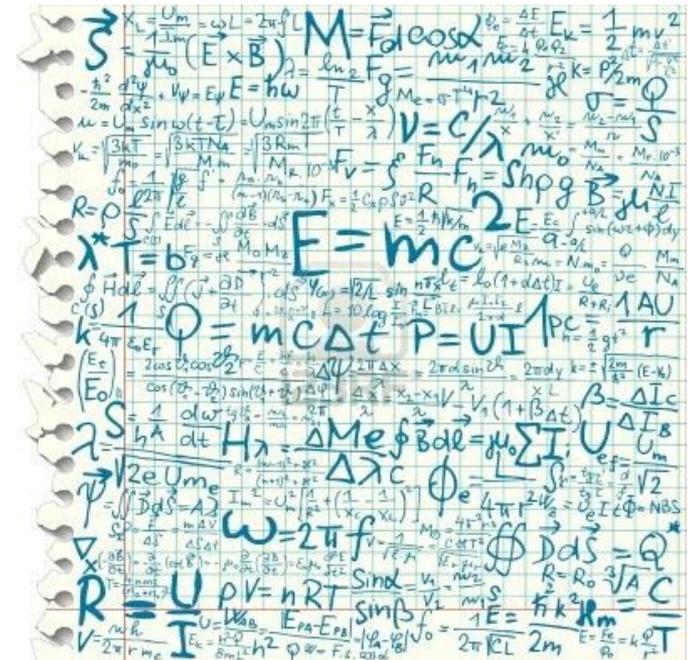
<sup>1</sup> Universität Hamburg – [ricardo.karam@uni-hamburg.de](mailto:ricardo.karam@uni-hamburg.de)

<sup>2</sup> Universität Halle – [olaf.krey@physik.uni-halle.de](mailto:olaf.krey@physik.uni-halle.de)



# Role of equations in physics lessons

- Calculation tools (*Plug-and-Chug*)
- Description of empirical regularities
- Belief in the authority of formulas
- Lack of understanding (Why?)
- Teachers are an **essential** part of the solution!





# Seminar for teacher students



Dez 2012  
Pilotierung UHH

Mär-Mai 2013  
Kurs U Helsinki

Okt-Dez 2013  
Kurs TU Dresden

Apr 2014  
Datenanalyse

## Epistemological Motivation

In the further development of science, **we want more than just a formula**. First we have an observation, then we have numbers that we measure, then we have a law which summarizes all the numbers. **But the real glory of science is that we can find a way of thinking such that the law is evident** (Feynman, 1964)

## Pedagogical Motivation

PCK includes the **most useful forms of representation, the most powerful analogies, illustrations, examples, and demonstrations** [...] Since there are no single most powerful forms of representation, the teacher must have at hand a **veritable armamentarium of alternative forms of representations** (Shulman, 1986)



## Part 1: History and Philosophy (6 h)

**Text 1:** Consequences of the mathematization of physics (1700-1900):  
Social, Epistemological und Ontological (Gingras)

**Text 2:** Significance of some basic mathematical concepts (multiplication, function, real and complex numbers) for Physics (Bochner)

**Text 3:** The interplay of physics and mathematics (Feynman)

- Role of Mathematics in Physics and vice-versa;
- Mathematics as a language and/or reasoning
- Different thinking of mathematicians and physicists

## Part 2: Explanations repertoire (4 x 3 h)

$y = \frac{(v_e + v_a)}{2} t$

$y = \frac{1}{2} g t^2$

$\left[ \frac{y}{g \cdot t^2} \right] = 1$

$g = \frac{d^2 s}{dt^2} = const.$

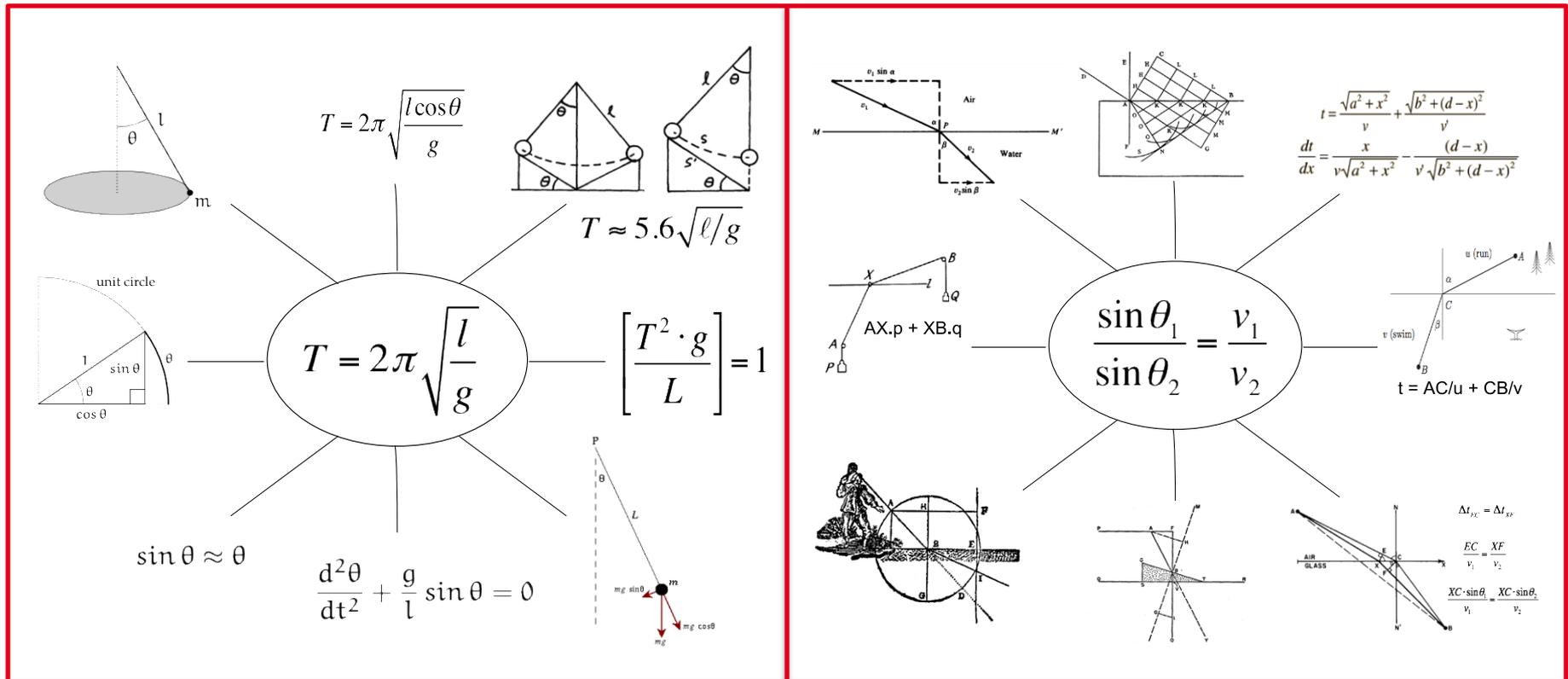
$S_n$   
 $\Delta S_n$   
 $\Delta(\Delta S_n)$

$a_z = \frac{v^2}{r}$

$\vec{a} = \frac{d^2 \vec{r}}{dt^2} = -\omega^2 \vec{r}$

$\left[ \frac{a \cdot r}{v^2} \right] = 1$

## Part 2: Explanations repertoire (4 x 3 h)



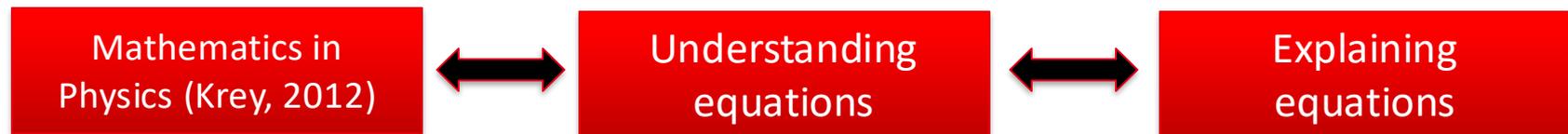


## Research questions

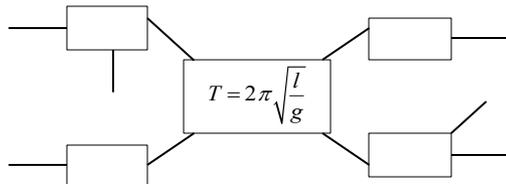
- 1) Are there changes in the students' ideas about:
  - 1.a) **The relationship between physics and mathematics?**
  - 1.b) **The role of formulas/derivations in physics lessons?**
  
- 2) How did the students use the learning opportunities of the seminar?  
(**Case study basis**)

## Data (written)

### 1) Questionnaire



### 2) Mind maps and explanations repertoire



One of your students asked : “Why does the equation have this structure?” Please sketch **different ways** to answer this question. The more, the better.

### 3) Reflection portfolio

10 min in the end of each meeting – Register the learning process

- What did you learn today? What did you dislike? Which questions were left unanswered?

## Data (Video)

### 4) Recordings of the seminar

- Discussions: Learning difficulties; Pre-requisite; Curriculum
- Reasons for preferring one derivation

### 5) Interviews with 4 students



- Justify answers (written) and manifestations in class
- Evaluate the seminar regarding contributions for future work as teachers



## The case SDX78 - Michael

- Emphasis in high-school: Physics & German; 2<sup>nd</sup> topic: History
- Small interest (2) for theoretical physics
- **Negative attitude** towards mathematics:

“Mathematics brings students to despair and is responsible for many conflicts in German families” (Pre)

- After questionnaire (Pre) in the **reflection portfolio**:

“I found the questions about how to explain equations quite difficult. I mean, we often calculate with them, but rarely know where they come from”

## Role of mathematics in physics

### Metaphor (Pre)

*Mathematics is for physics like...*

- *a foothold to study nature*
- *impossible to get along without it*
- *barrier: prevents normal people from understanding physics*

### Metaphor (Post)

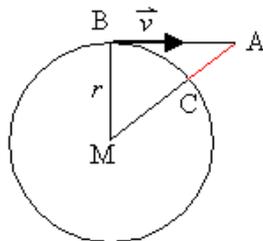
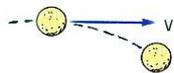
*Mathematics is for physics like...*

- *a language to categorize natural phenomena*
- *a structure that provides new knowledge*

## Deriving formulas

The function of derivations of formulas in physics is to show that it is ok to use them. (Pre: Strong agreement; **Post: Slight disagreement**)

*Interview: “We often asked ourselves: What is actually behind this formula? The most important thing was to **learn to ask these questions**; where does this and that come from? Derivations are very important for this purpose [...] My favorite formula is now the centripetal acceleration: I know where it comes from, I know what is behind it and I can derive it in a meaningful way. And that is when we really understand a formula.”*



$$\overline{BA} = v \cdot \Delta t \quad \overline{AC} = \frac{1}{2} a_z \cdot \Delta t^2$$

$$r^2 + (v \cdot \Delta t)^2 = \left( r + \frac{1}{2} a_z \cdot \Delta t^2 \right)^2$$

$$v^2 = r \cdot a_z + \frac{1}{4} a_z^2 \cdot \Delta t^2$$

for small  $\Delta t$

$$a_z = \frac{v^2}{r}$$

## Explaining $a = v^2/r$ in physics lessons

### $a = v^2/r$ (Pre)

- 1) Meaning of each symbol
- 2) Analyze what happens when one magnitude varies
- 3) Explanation of the formula with drawings and examples
- 4) Discuss its use in everyday life

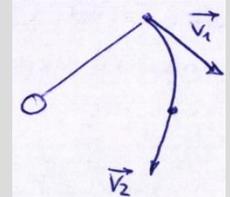
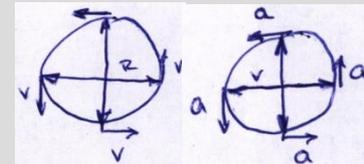
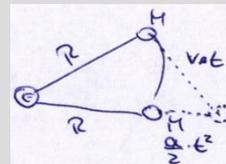
#### Exp repertoire:

$$a = v/t \text{ und } v = s/t ; a = v^2/s \text{ (s = r)}$$

### $a = v^2/r$ (Post)

- 1) Example (Moon around Earth)
- 2) Problem: What is the earth were not there?
- 3) Derivation using Pythagoras
- 4)  $a$  to the center ( $F_G$  to Earth)

#### Exp repertoire: 3 Derivations





## Conclusions & Perspectives

- Equations in physics lessons: from describe/calculate to explain/understand
- Seminar for teacher students: HPS + Explanations' repertoire
- Case-study (SDX78)
  - Better attitude regarding mathematics in physics
  - Connection between understanding and deriving equations
  - From describe/calculate to derive equations in physics lessons
- General evaluation of the seminar and improvements
- Understanding and explaining equations in school: "Didactics of Formulas"